I.2 EXTENDED VIDEO: Composite Functions

In this lesson we look at the integration rule for composite functions. Note that in this first example we have a function to a power of 3. In order to integrate you must have the derivative of the function present in the integrand. The derivative is $6x$ and it is in the integrand so you can integrate by raising the function to one greater power and dividing by the new exponent.

$$\int 6x(3x^2 + 5)^3 \, dx = \frac{(3x^2 + 5)^4}{4} + C$$

In this next example the derivative is $6x$ so we insert a 6 and outside the integral symbol we insert $1/6$. We can do this because $6x \times 1/6$ is 1 so we are multiplying by 1 which does not affect the value of the original problem. It just allows us to proceed with the integration.
\[
\int x(3x^2 + 5)^3 \, dx = \frac{1}{6} \int 6x(3x^2 + 5)^3 \, dx = \frac{1}{6} \frac{(3x^2 + 5)^4}{4} = \frac{1}{12}
\]

In this example we have the components to proceed with the integration.

\[
\int (5x + 4)^9 \cdot 5 \, dx = \frac{(5x + 4)^{10}}{10} + C \quad \text{or} \quad \frac{1}{10} (5x + 4)^{10} + C
\]

Note in the next example the factor of 5 must be inserted to proceed.

\[
\int (5x + 4)^9 \, dx = \frac{1}{5} \int 5(5x + 4)^9 \, dx = \frac{1}{5} \frac{(5x + 4)^{10}}{10} = \frac{(5x + 4)^{10}}{50}
\]
In this example we pull the 4 outside the integrand and insert the 10 that is required.

\[ \int 4x(5x^2-8)^6 \, dx = \frac{4}{10} \int 10x(5x^2-8)^6 \, dx = \frac{4}{10} \frac{(5x^2-8)^7}{7} = \]

In this example we pull the 4 outside the integrand and insert the 6 that is required.

\[ \int (2x^3-1)^4 \, 4x^2 \, dx = \frac{4}{6} \int (2x^3-1)^4 \, 6x^2 \, dx = \frac{2}{3} \frac{(2x^3-1)^5}{5} = \]

If a radical is present first change this to exponential form. In this next example we also need to make an adjustment by inserting a factor of 3.
\[
\int x^2 \sqrt{x^3 + 5} \, dx = \int x^2 (x^3 + 5)^{1/2} \, dx = \frac{1}{3} \int 3x^2 (x^3 + 5)^{1/2} \, dx
\]

In this final example we need to make an adjustment by inserting a factor of 3.

\[
\int \frac{x^2 + 5}{(x^3 + 15x)^2} \, dx = \frac{1}{3} \int 3(x^2 + 5)(x^3 + 15x)^{-2} \, dx = \frac{1}{3} \frac{x^3}{x^3 - 15x} + C
\]